Analysis and Modelling using Machine Learning of the Interconnection among Mathematics Anxiety, Mathematical Errors and Working Memory of Middle School Students

Synopsis

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1.0 INTRODUCTION

As released by The Australian Council for Educational Research (ACER) on December 15, 2011, the results of the Program for International School Assessment (PISA) reveal that only 15% of students of India (Tamil Nadu & Himachal Pradesh) are proficient in mathematics which is not an appreciable percentage. Why 85% are incompetent? The reason might be the anxiety for mathematics in students, their inability to process small amounts of information or committing errors while solving problems in mathematics. The knowledge of mathematics and skill provide a key for entry into the rapidly changing technological world (Leonard, 2014). But unfortunately, there are many children and adults who feel anxious whenever they face any maths - related situation. Such a feeling of stress while confronting any mathematics problem is called maths anxiety (Sokolowski & Ansari, 2017). This anxiety utilises some of the working memory. As a result, not enough working memory is left for solving the maths problem and this in turn makes the individuals feel anxious (Ramirez, et. al., 2012).

Mathematics anxiety

The feeling of stress, tension and mental disorganisation that is faced when an individual is in mathematical problem-solving situations is mathematics anxiety (Zeidner, 2010). Some individuals when presented with maths stimuli show avoidance behaviour, "increased physiological reactivity" (Dew, Galassi, & Galassi, 1984, Faust, 1992) and negative emotions. Such are said to have mathematics anxiety (Hopko et. al., 2003). This fear of mathematics is common among people and can also limit the performance of students in mathematics related situations (Zhang, Zhao & Kong, 2019). Some Australian students stated that the tension which individuals feel while completing any mathematical task is mathematics anxiety (Thomson, DeBortoli & Buckley, 2013). Mathematics anxiety is a lack of confidence among individuals when unable to solve any mathematical problem (Severs, 2021). Mathematics anxiety is a "negative cognitive reaction" of someone when dealing with mathematics (Holmes, 1991). "Individuals who consistently experience mathematics anxiety when engaging with mathematics are more likely to avoid mathematics subjects, courses and careers" (Buckley, n.d.). Mathematics anxiety needs to be studied as it negatively impacts students' performance, grades and knowledge (Ashcraft & Krause, 2007; Suinn, Taylor & Edwards, 1988; Ramirez, et. al., 2012). Trujillo and Hadfield (1999) consider mathematics anxiety as "a discomfort situation" which arises when an

individual is involved in any mathematical task which is meant to threaten the self-esteem of the individual. People experiencing mathematics anxiety are confused, feel unsafe, face physical variations like problems in breathing, tightening of muscles, etc. when stuck in a mathematical task (Arem, 2003). Students facing such anxiety and fear for mathematics accept that they are unable to solve the problems related to mathematics and as a result run away from mathematics lessons and the subject as a whole (Anindyarini & Pahar, 2019). Students of every level (primary school to university students) may face mathematics anxiety (Khasawneh, Gosling & Williams, 2021). Many students who suffer from mathematics anxiety lack confidence in confronting mathematics and tend to opt for very less compulsory mathematics courses and this limits their career (Khasawneh et. al., 2021). Arem (2003) calls mathematics anxiety as "a three - fold association of poor test-taking strategies, poor test preparation and psychological pressure". Certain goals are set up by the National Council of Teachers of mathematics for the curriculum development and evaluation of the subject at all levels. Like, making the students understand the use of mathematics techniques, solve mathematical problems, use abstractions to perceive relationships and enjoy mathematics. Making efforts to accomplish these goals can reduce the fear of mathematics in students (Teaching of mathematics, 2006, p. 2 & 3).

Mathematical Errors

While learning mathematics, students often make errors (Warshauer, 2015). The common mistakes made by students while facing difficulties in solving mathematical problems are termed as mathematical errors (Herholdt & Sapire, 2014; Ketterlin-Geller & Yovanoff, 2009; Rong & Mononen, 2022). The number of errors made by students provides an idea to whether the students have understood the content or not and up to what extent have they understood. Once these errors are identified, the source of errors can also be investigated (Winarso & Toheri, 2021). The errors made by students become a barrier in their learning process. If a teacher wants to know the trouble a student is facing in learning mathematics, she should focus on the errors committed by students in some primary topics of the subject (Taylor, 1999). The process of reviewing a student's work is error analysis (Lai, 2012). Timely analysis of these errors is a must. One of the rules to analyse these errors is Newman's error analysis. This procedure has been applied around the globe like India, Malaysia, Australia, Thailand, etc. (Suyitno, 2015). It analyses the errors committed by the students from the beginning (when the question is presented) till the end.

According to Newman, a number of consecutive hurdles are faced by an individual when he/she tries to attempt a standard mathematics question. The obstacle is a type of error in Newman's Error Analysis (Jha, 2012). In Newman's process, when solving any mathematical problem, students hold a support of many factors which help them to reach the correct answer. The method is based on the fact that students face two types of hurdles which creates a hindrance in reaching the correct answer. These hurdles are: i. Difficulties in language fluency and abstract understanding and reading of problems. ii. Difficulties in processing mathematical problems which include transformation, process skills, and encoding answers (Jha, 2012).

Newman has given five levels of error analysis (Chusnul, Mardiyana & Retno, 2017).

i. Reading level

ii. Comprehension level

iii. Transformation level

iv. Process skills level

v. Encoding level

In the first level i.e., reading, it is analysed whether students can read the given problem or not. In the second level, the comprehension level, the error might occur in the understanding of the problem; the third level, i.e., transformation level, it is noticed whether the student has selected the appropriate mathematical operation and procedure or not; in the fourth level, process skill level, it is analysed whether the student can perform the mathematical procedure and calculation accurately or not; and in the last level i.e., encoding level, error might be seen in the appropriate representation of the final answer. Once these errors are identified, the required remediation can be provided.

Working Memory

The concept of working memory was extracted from that of short-term memory (the temporary storage of small amounts of material over a short period of time) (Baddeley, 2010). The term working memory was introduced by Miller, Galanter and Pribram in 1960 in a book titled 'Plans and Structure of Behaviour' but was used primarily by Atkinson and Shiffrin in 1968 in their paper 'Human memory: A proposed system and its control processes'. "A limited capacity part of the human memory system that combines the temporary storage and manipulation of information in the service of cognition" is working

memory (Baddeley & Hitch, 2010). Working memory refers to the capacity to hold information for a short period of time in memory along with the processing of the information (Mirsky et al., 1995). What lies to the central functioning of the mind is working memory. It is correlated with various other individual abilities like intelligence and scholastic achievement and has a link with the sensory processes of a person (Burmester, 2017). “The temporary storage” and “manipulation” of information from the past to reach the present goals is working memory (Halliday, Torres & Trujillo, 2015). Working memory is concerned with two functions, namely, “processing” and “storage”, which are probably the essentials for certain aspects of cognitive abilities, like arithmetic calculations, logical reasoning and language processing, etc. (Wen, 2021). Bistami and Kowalcyzk (2021) addressed majorly three components of working memory i.e., “encoding, storing and retrieving of information”. A “multicomponent system” which involves manipulation of information for complex cognitive usage (Baddeley and Hitch, 1974; Baddeley, 1996, 2000; Chai, Hamid and Abdullah, 2018) is working memory. It helps us to keep a track of what we are doing from time to time and to focus on our actions adopted by us to solve problems (Logie, Camos & Cowan, 2020).

Machine Learning

A recent trend in machine learning i e., machine learning - based modelling is being applied in research for explaining, predicting and describing a person’s behaviour (Yang, et al., 2021) It is bringing transformations in education and research. The pioneer of machine learning Arthur Samuel (1959) defined machine learning as “a field of study that gives computers the ability to learn without being explicitly programmed.” After some time, Tom Mitchell (1997) gave a modern definition of machine learning i.e., Machine learning is the study of computer algorithms that allow computer programs to automatically improve through experience. This definition was more explained as “A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P, if its performance at tasks in T, as measured by P, improves with experience E.” It is the ability of computer systems to improve automatically by learning and applying the gained experiences (Valihura & Chornyi, n.d.). Similar to the students learning to distinguish between a cat and a dog, computers are learning patterns and certain rules to do this distinction. One need not feed the data or coding manually, rather let the computers learn by themselves. This is called machine learning (Hilbert, et al., 2021). Machine

learning is a method of data analysis in which algorithms are used to build relationships and to make predictions and estimations (Davison, n.d.). Machine learning acts as a game changer in the field of education with the following benefits like, for predicting students' performance, for grading students fairly, for organising content effectively, for predicting an appropriate path for career, for suggesting better learning paths (Jagwani, 2019). Machine learning demands for a large volume of data to a machine so that it can learn, classify data or make predictions.

There are three types of machines learning *viz.* Supervised learning, unsupervised learning and reinforcement learning. When a machine is provided with a known dataset and allowed to analyse, then the machine produces an "inferred function" to predict the output. Such a learning is known to be supervised learning (Selig, 2022). In unsupervised machine learning, the machine is fed with an unlabelled dataset wherein the machine itself has to classify the data and find the hidden pattern without any human interference, i.e., one doesn't need to provide output to the model (Gupta, 2022). A type of machine learning which is similar to human learning is reinforcement learning. In this type, the machine learns or sets up the algorithm by interacting with the environment and acquiring the positive and negative rewards from it (Coursera, 2022). The machine learns from new situations using the trial-and-error method (Friday, 2019). Machine learning will bring novel opportunities in the field of education and might shrink the gap amidst the teachers and students (Jagwani. 2019).

1.1 EMERGENCE

Maths anxious individuals are less efficient and struggle in updating their working memory (Pelegrina, Galiano, Puga & Linares, 2020). Research has also revealed that individuals who are anxious in mathematics experience an interruption in their working memory which in turn leads to a reduction in the capacity of the individual to solve problems related to mathematics and are also prone to more error-making (Ashcraft & Kirk, 2001; Eden, Heien & Jacobs, 2013; Ma, 1999). Those who are anxious in mathematics observe a decrease in their short-term learning and they move away from mathematics opportunities due to a hindrance in their long-term learning (Buckley, Reid, Good, Lipp & Thomson, 2016). Findings by Passolunghi, et al. (2016) suggests that a person with high mathematics anxiety shows weak performance in working memory tasks and also students, who had high mathematics anxiety, were unable to retrieve facts and information. The general anxiety affects the individuals at an

early age as it undermines their working memory and later on, a similar effect of mathematics anxiety was also observed (Pellizzoni, Cargnelutti, Cuder & Passolunghi, 2021). Identifying students' mathematical errors is the preliminary step for providing them with corrective measures in solving the mathematical problems (Lai, 2012). Students who are highly maths anxious are more prone to committing errors in complex arithmetic problems. This results in their poor mathematics performance (Sheffield & Hunt, 2007). Research shows that when mathematics anxiety increases, the mathematics performance decreases and the vice versa is also true (Carey, Hill, Devine & Szűcs, 2016). One might conclude that decrement in performance is visible as students commit more errors as compared to those whose performance is good. Male students perpetrate more errors than female students (Subekti & Zuhrotunnisa, 2021) whereas female students are more anxious in mathematics than male students (Devine, Fawcett, Szucs & Dowker, 2012; Mier, Schleepen & Berg, 2018). The above findings might lead to the conclusion that male students being less anxious in mathematics commit more errors while female students commit less errors being more anxious in mathematics. This indicates an inverse relation between mathematics anxiety and mathematical errors. So, we can't predict what level of mathematics anxiety leads to what type of errors. May be highly anxious students are unable to comprehend, or maybe they are inefficient in applying appropriate formulas or any other. We can't predict without any research. Therefore, machine learning comes into action. Machine learning in education is an emerging concept. Making use of machine learning in teaching - learning process, makes this efficient and productive (Lyzanets, 2022). Machine learning algorithms might assist schools and faculties to help students to become successful and reach out to students as early as possible (Kučak, Juričić & Đambić). As per Brooks and Thompson, 2017, machine learning is used to identify students who are going down in their academics programming and also being used to predict achievement of students. The predictive models built by machines are used to predict whether the students will achieve success in time or will it be delayed based on various factors of their daily routine (Aguiar, et al., 2015). Prediction will also be made in the present research using machine learning along with exploring answers to questions as follow:

- What are the types of mathematical errors committed by middle school students?
- What is the level of mathematics anxiety of middle school students?
- What is the level of working memory of middle school students?

- How are mathematical errors and mathematics anxiety related?
- How are mathematical errors and working memory related?
- How are mathematics anxiety and working memory related?
- How does working memory affect the frequency of errors committed by middle school students?
- How does mathematics anxiety affect the frequency of errors committed by middle school students?
- How does mathematics anxiety affect the working memory of middle school students?
- What is the interconnection among working memory, mathematics anxiety and mathematical errors committed by middle school students?
- Which type of pattern of the interconnection among working memory, mathematics anxiety and mathematical errors will be generated? This pattern will be based on the answers of the following questions:
    - Which level of mathematics anxiety and working memory gives rise to which type of error?
    - What level of working memory and type of mathematical errors committed by middle school students leads to which level of mathematics anxiety?
    - Which level of mathematics anxiety and type of mathematical errors committed by middle school students leads to what level of working memory?
- What type of model will be developed by machine once these patterns are identified?

1.2 JUSTIFICATION

The application of machine learning in education is amplifying every day. Students and teachers both are receiving the benefits as performance of students can be predicted, time to time testing of the students can be done, fair grading is possible, etc. Machine learning is also being used to build relationships among variables in order to predict one, using the others. In the present study, machine learning will be used to predict any one from mathematics anxiety, mathematical errors and working memory based on the other two. Students commit various errors in mathematics and this might affect their working memory and as a result they might be anxious towards mathematics. We need to build such a model which can be applicable in future as well, so as to predict which type of error the students will commit based on their working memory and mathematics anxiety scores. This model will be developed through machine learning.

Algorithms will be used to make the machine learn. Based on the data collected, the machine will decide which type of model best suits the data. It might be a decision tree or a mathematical equation or any other pattern. The data will be divided into two segments randomly: Training and testing. Whatever the machine gives as an output for the training data, it will be validated for the testing data. The machine will use an iterative approach to learn from the data provided.

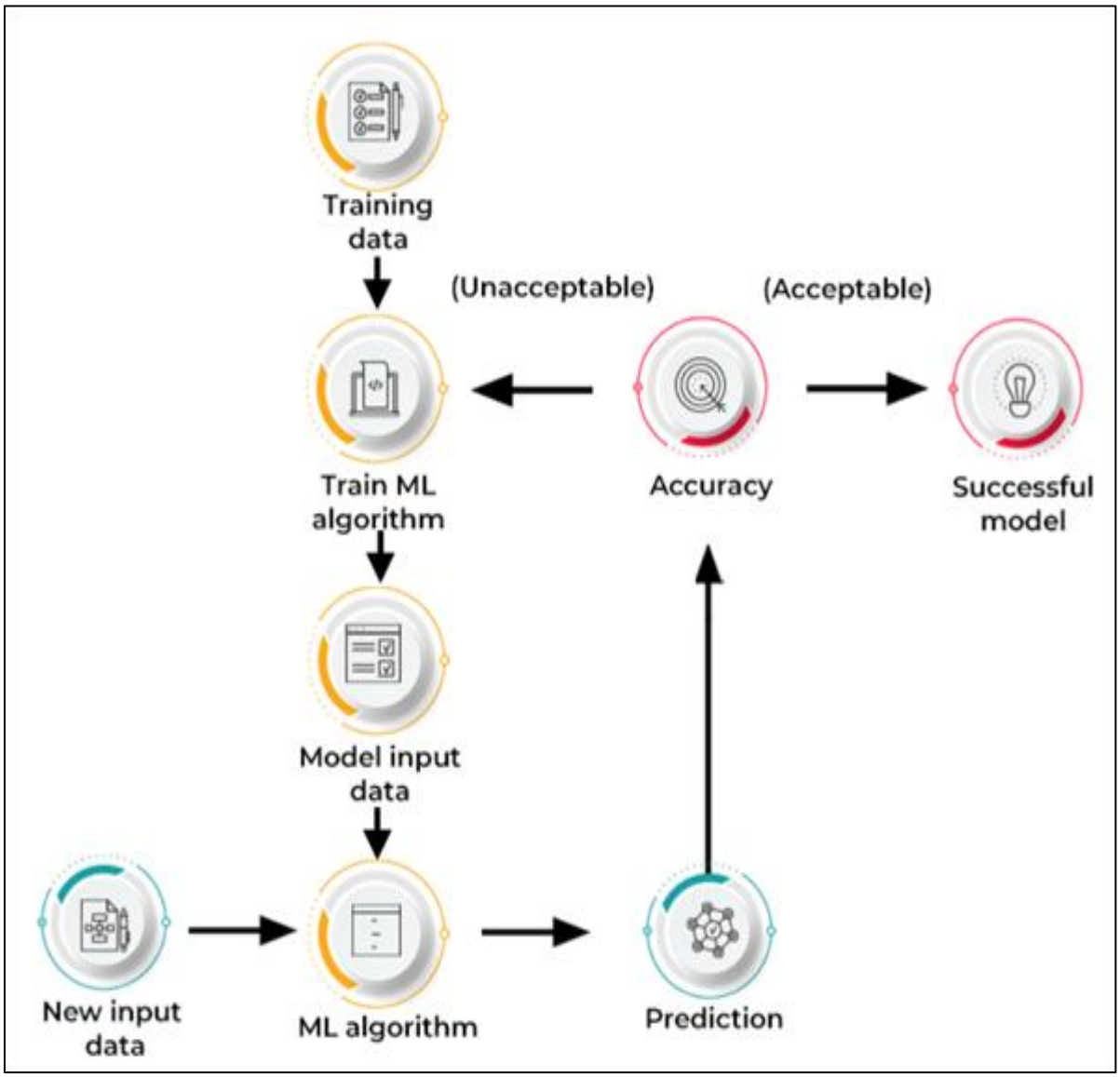

Figure 1. How does Machine Learning Work?
(Source: https://www.spiceworks.com/tech/artificial-intelligence/articles/what-is-ml/)

In figure 1, the training data for the present research will be fifty percent of the data collected i.e., scores of mathematics anxiety and working memory and the types of mathematical errors committed by middle school students. The model input data will be the rest of the fifty percent data collected. As perfectly as the findings fit into the model, it can further be used for any new data. This is necessary in order to

prepare the students to fight against the hurdles which appear in their lives related to mathematics. Bringing Machine learning into education might help to eradicate the errors committed by students in mathematics and this might encourage them to select a career in the same field. For this, the errors must be known prior. This is possible by setting up a model which will predict the errors. And this prediction will be made by machine learning. Once the errors are known, the teachers and the students can work collaboratively to rectify them. This might also improve the academic performance of the students. Mere finding the relationship among all three, will not lead to any contribution in students' performance in future. Rather we need to set up a permanent model for the same. And this little contribution can do big wonders in the lives of the students. Therefore, analysis and modelling of the interconnection among mathematics anxiety, mathematical errors and working memory is necessary.

1.3 STATEMENT OF THE PROBLEM

Analysis and modelling using machine learning of the interconnection among mathematics anxiety, mathematical errors and working memory.

1.4 FUNCTIONAL DEFINITION OF THE TERMS USED IN THE STUDY

1. *Mathematics anxiety*:

Mathematics anxiety is a state of mind which involves avoidance behaviour, low - grade performance, negative perceptions and increased nervousness when presented with maths stimuli (Hopko, Mahadevan, Bare & Hunt, 2003). It is an emotional response which is elicited by mathematics in particular (Carey, Hill, Devine and Szűcs, 2017).

2. *Mathematical errors:*

The ubiquitous errors made by students on the basis of the difficulties experienced by them when they deal with mathematical problems are referred to as mathematical errors (Rong & Mononen, 2022).

3. *Working memory:*

Working memory is the capacity to store and manipulate retained information. It is necessary for completing complex cognitive tasks, such as making decisions, reasoning, reading and learning (Breznitz, 2020).

4. *Machine Learning:*

Machine learning (ML) is a discipline of artificial intelligence (AI) which allows the machines to

automatically learn from the data provided and also from the past experiences along with the recognition of the patterns so as to make predictions with minimal human intervention (Kanade, Aug., 2022 & Selig, Mar., 2022).

1.5 OBJECTIVES OF THE STUDY

1. To study the mathematics anxiety of middle school students.
2. To study the mathematical errors committed by middle school students.
3. To study the working memory of middle school students.
4. To find the relation between mathematics anxiety and mathematical errors committed by middle school students.
5. To find the relation between working memory and mathematical errors committed by middle school students.
6. To find the relation between mathematics anxiety and working memory of middle school students.
7. To study the role of mathematics anxiety in predicting the mathematical errors committed by middle school students.
8. To study the role of working memory in predicting the mathematical errors committed by middle school students.
9. To study the role of mathematics anxiety in predicting the working memory of middle school students.
10. To study the interrelationship among mathematics anxiety, mathematical errors and working memory of middle school students.
11. To develop a mathematical model using machine learning:
    - 11.1. To predict the mathematical errors committed by middle school students on the basis of their mathematics anxiety and working memory.
    - 11.2. To predict the mathematics anxiety on the basis of the mathematical errors committed by middle school students and their working memory.
    - 11.3. To predict the working memory on the basis of the mathematical errors committed by middle school students and their mathematics anxiety.

1.6 HYPOTHESES OF THE STUDY

$H_0$1: There will be no significant relationship between mathematics anxiety and mathematical errors committed by middle school students.

$H_0$2: There will be no significant relationship between working memory and mathematical errors committed by middle school students.

$H_1$3: There will be a significant relationship between mathematics anxiety and working memory of middle school students.

$H_0$4: The mathematics anxiety of middle school students does not significantly predict the mathematical errors committed by them.

$H_0$5: The working memory of middle school students does not significantly predict the mathematical errors committed by them.

$H_1$6: The mathematics anxiety of middle school students significantly predicts their working memory.

$H_0$7: There will be no significant interrelationship among working memory, mathematics anxiety and mathematical errors committed by middle school students.

The following hypotheses will be tested once the model is developed through machine learning:

$H_0$8: The working memory and the mathematics anxiety does not significantly predict the mathematical errors committed by middle school students.

$H_0$9: The working memory and type of mathematical errors committed by middle school students does not significantly predict their mathematics anxiety.

$H_0$10: The type of mathematical errors committed by middle school students and the mathematics anxiety does not significantly predict their working memory.

1.7 VARIABLES OF THE STUDY

The study variables for the present research are: mathematics anxiety, mathematical errors committed by middle school students and working memory. These variables will be called as features when machine learning is being applied.

1.8 PROCEDURE OF THE STUDY

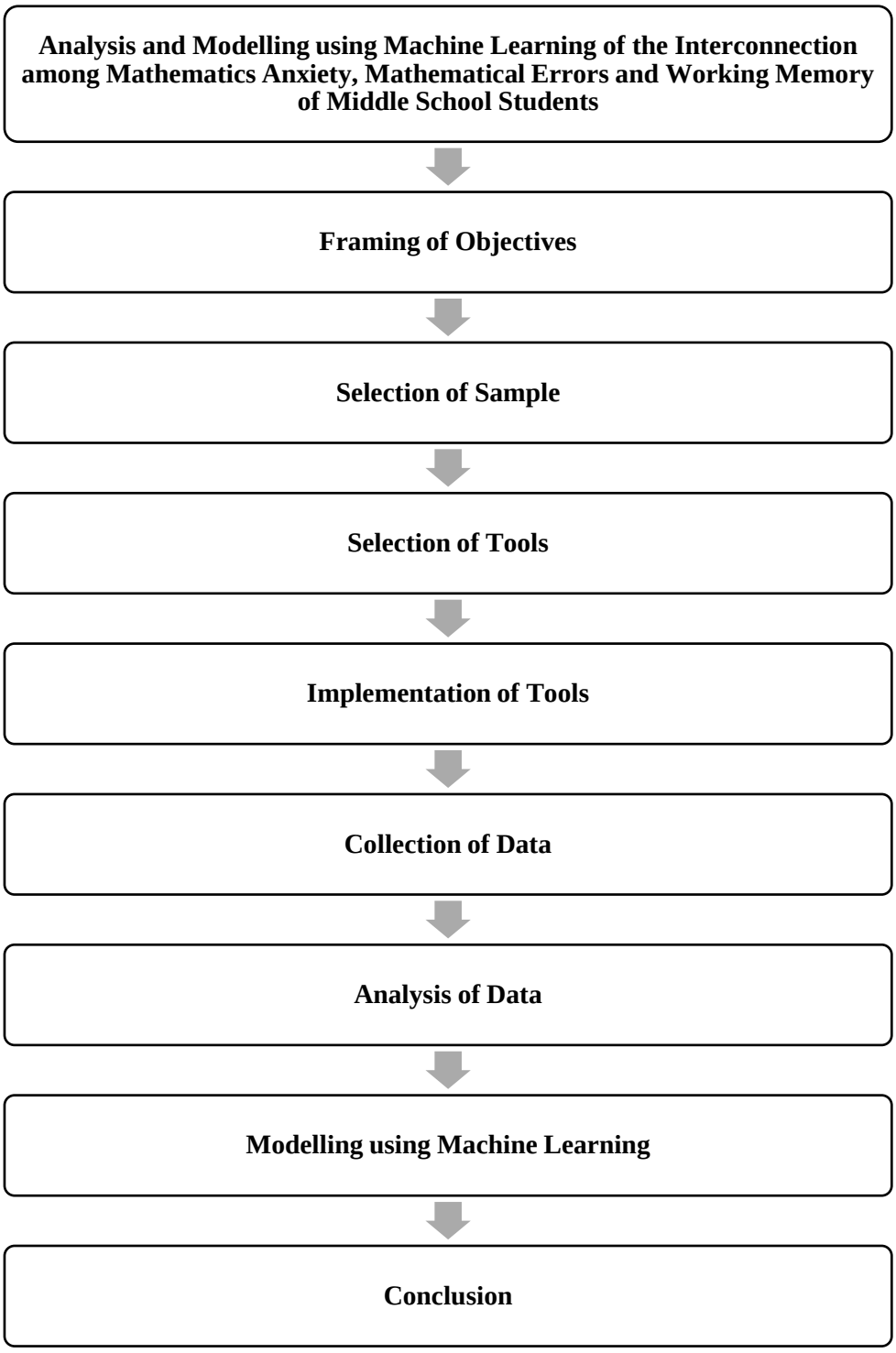
Analysis and Modelling using Machine Learning of the Interconnection among Mathematics Anxiety, Mathematical Errors and Working Memory of Middle School Students
Framing of Objectives
Selection of Sample
Selection of Tools
Implementation of Tools
Collection of Data
Analysis of Data
Modelling using Machine Learning
Conclusion

Figure 2. Procedure of the Study

1.9 METHOD OF THE STUDY

The present research will be conducted in two phases. In the first phase, the research methodology to be adopted will be a descriptive survey methodology as the interconnection among mathematics anxiety,

mathematical errors and working memory of middle school students will be studied. In the second phase, the study will be development research as a model will be developed from the training data through the machine and will be applied to the testing data. This will result in the modelling of the relationship.

1.10 POPULATION OF THE STUDY

The population of the present study is the middle school students (Class VI – VIII) of DEI Board. The age group of the population is 10 - 13 years.

1.11 SAMPLE OF THE STUDY

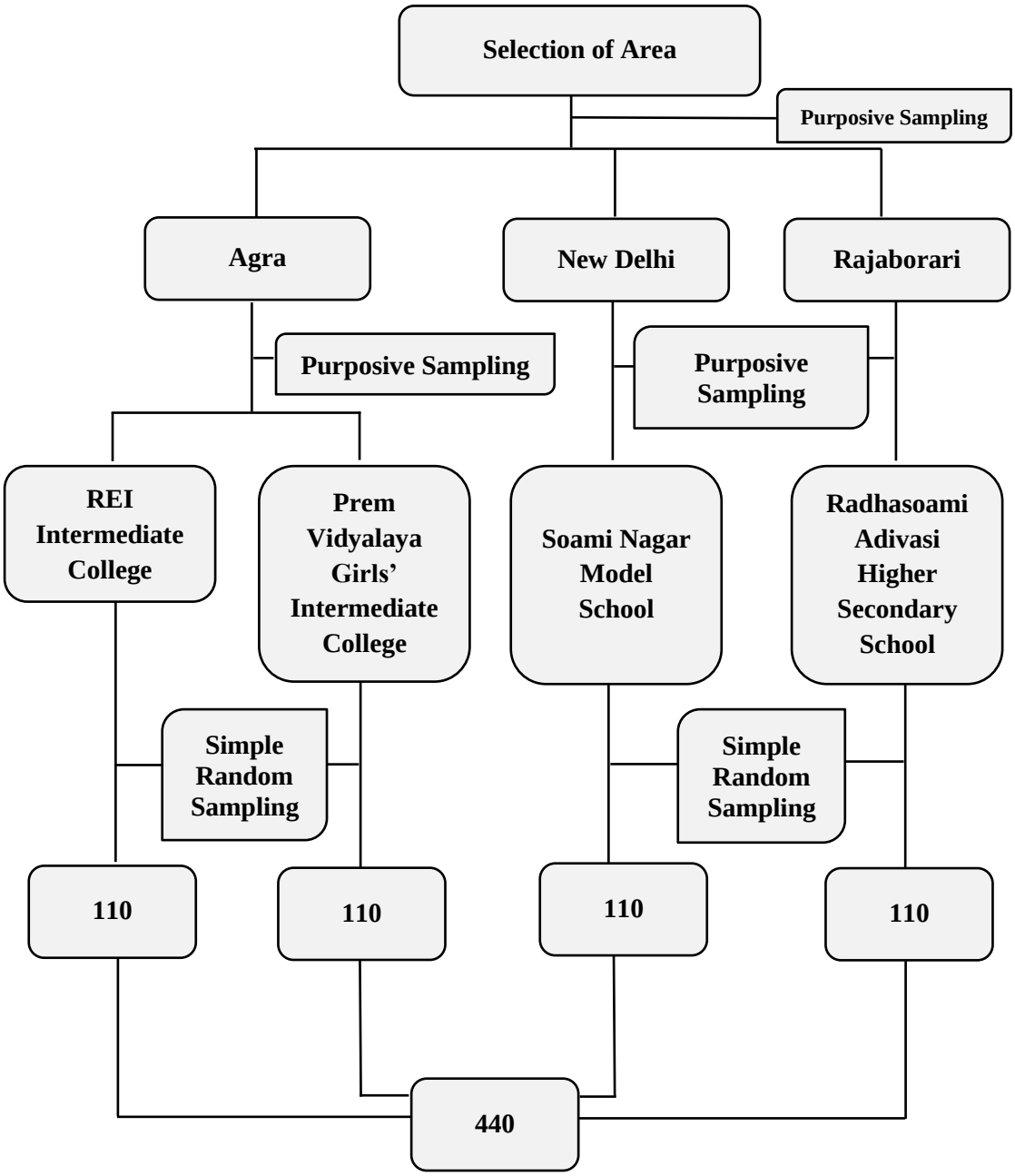

Figure 3. Sample of the Study

The sample area selected for the present study is the schools from Agra, New Delhi and Rajaborari. Two schools from Agra, one from New Delhi and one from Rajaborari have been selected. Agra, New Delhi and Rajaborari are chosen purposively by the researcher as purposive sampling means the sampling based on the judgement of the researcher (Dudovskiy, 2022). Therefore, it was researcher's judgement to select these areas. The reasons for selecting schools of Agra, New Delhi and Rajaborari specifically are: Firstly, the comparison when made on common grounds is more reliable than otherwise. So, only DEI board schools are selected. Secondly, a comparison between urban and tribal areas using the model being built through machine learning can also be made. Four schools are selected in all and 110 students from the middle school level of each school are chosen. This adds up to the total sample size of 440 students.

1.12 TOOLS OF THE STUDY

1. Mathematics Anxiety

The tool to be used for measuring mathematics anxiety of middle school students is mAMAS (Modified Abbreviated Math Anxiety Scale) by Carey, Hill, Devine and Szűcs developed in the year 2017. The reliability of the tool was assessed using ordinal alpha as well as Cronbach alpha. The reliability calculated through ordinal alpha is 0.89 and that through Cronbach alpha is 0.85. The tool consists of nine items. It is a 5 - point Likert scale to indicate how anxious the subject feels during situations involving maths (1 = low anxiety to 5 = high anxiety).

2. Mathematical Errors

Self-constructed test will be used to assess errors by observing the solutions of questions (test items) step-by-step. The test will be prepared on the basis of the related literature and MLOs (Minimum Learning Outcomes).

3. Working Memory

The CogniFit Assessment designed by Shlomo Breznitz, 2020 includes a comprehensive battery of tasks. Each of these exercises has been carefully selected and been categorised to measure basic brain functions (23 cognitive skills) including working memory. The working memory tool holds an internal consistency (calculated through Cronbach alpha) of 0.85 and the test - retest reliability of 0.696.

1.13 STATISTICAL TECHNIQUES

Following statistical techniques will be used in the study:

Descriptive Statistics

Mean: It will be used as a measure of central tendency.

Standard deviation: It will be used as a measure of variability.

Inferential Statistics

Multiple Correlation: It will be used to determine the relationship among mathematics anxiety, mathematical errors and working memory of middle school students.

Analysis and Modelling

Machine Learning: It will be used to develop a model for the interconnection among mathematics anxiety, mathematical errors and working memory of middle school students.

Graphical Representation

Data will be presented in graphs with the help of graphical representation of scores.

1.14 SIGNIFICANCE OF THE STUDY

The present research is about analysing the interconnection among mathematics anxiety, mathematical errors and working memory of middle school students and also discovering a model of this interconnection via machine learning. This study will help in building such a model which might prove effective in predicting one over the other two. Once the teacher or management knows about the anxiety and working memory of students, they might get an insight as to what type of errors the students will commit. The interconnection being established in this research might lead to the discovery of such a model which might be pertinent for remote and urban area students and this would eradicate the limitation of the model on the students in one area only. This study might also benefit students, teachers and management to know which type of errors students commit, which level of mathematics anxiety or how much working memory the students hold and if needed can also improve prior. Making the machine learn about the relationship among mathematics anxiety, mathematical errors and working memory will constitute a forever establishment of such a relationship. Students are very anxious especially in the subject of mathematics. They are afraid to opt maths as their mainstream subject and also for their career. This anxiousness might lead to the increment or decrement of the frequency of errors the students

commit depending on the relationship which will be obtained. So, this will guide the students as well as the teacher into which area to work. Mathematics anxiety and working memory are negatively related as a number of researches. So, this relationship remains consistent. In the present study, this relationship will also be tested.

Identifying errors at the middle school age is a must. Reducing anxiety and errors might raise the frequency of the students who might opt mathematics as their career. As soon as a student leaves behind the middle school the next stage which appears in front of him/her is the secondary stage. As per NEP, 2020, this is the stage wherein the students will be given a chance to select the subjects. And also, this is the time where the student starts developing an insight about their career regarding their subject choice. So, eradicating the fear, decreasing the frequency of errors in mathematics at this stage, i.e., middle school level, might lead to an escalation in the number of students picking mathematics as their career.

Working memory is an important aspect of an individual's life since childhood. The better the working memory, the sharper the brain will become or vice versa. This will help in understanding and processing any information received. So, an understanding of the working memory of students is also needed in order to reach the aim of getting more students in the career in mathematics. If the teacher holds a knowledge of the working memory of the students, she can work accordingly to improve the students. Also, this will give an idea as to how to deal with the mathematics anxiety of students (the previous research has already tested the relationship between mathematics anxiety and working memory).

So, bringing machine learning in education will support the teachers to handle the students' frequency of errors in mathematics and their mathematics anxiety and working memory. Not only teachers, but also students, parents, mentors, administration, counsellors, all will be benefited with this research once the modelling of the interconnection among mathematics anxiety, mathematical errors and working memory via machine learning is done.

1.15 DELIMITATIONS OF THE STUDY

The study is delimited to the middle school students of DEI Board of Agra, New Delhi and Rajaborari. It is bound to study the transformation, process skill and encoding errors only.

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